UK Mathematical Sciences - the current context in Facts and Figures

Working Draft Version 4

1. Overview and summary:

The first finding of the EPSRC International Review of Mathematics 2004[1] stated:

> *The mathematical sciences are increasingly playing a more central role in the development of advanced modern societies because frequently advanced technology relies on sophisticated mathematical content. Mathematical research has to be considered from this angle, and therefore must be developed ambitiously. It must be properly connected to advanced training to meet the needs for properly trained personnel for the future, in industry and in academia.*

This report summarises the context in which research and training in the mathematical sciences is currently carried out in UK higher education. The focus here is on research and postgraduate training, but we emphasise from the outset that this cannot be separated from undergraduate education, both of those studying for degrees in mathematics and of those many students studying other disciplines for which the mathematical sciences are a crucial language and tool.

The past 20 years have seen a great flowering of mathematics world-wide: Wiles' proof in the 1990s of Fermat's Last Theorem in Number Theory has been followed by Perelman's 2003 proof of the 1904 Poincaré conjecture, one of the Clay Mathematics Institute's $1 million Millennium Problems. The full impact of these major breakthroughs on science, technology and on mathematics itself may not be realised for decades. A (very) long-term payback from fundamental research is often characteristic of the mathematical sciences: examples include results in number theory which led to the RSA algorithm which now underpins the security of much electronic communication and commerce, and results of Radon in 1917 and Fritz John in 1938 which provided the theoretical basis for two and three dimensional tomography respectively, over 40 years later[2].

Key points from the following pages are:

- the scale of UK higher education activity in the mathematical sciences has increased substantially over the last decade;
- (as regards research at least), this expansion has been accompanied by a concentration into fewer centres;
- a high proportion of YK mathematical sciences research is of top international quality, and this research is geographically widely based;
- undergraduate numbers in the mathematical sciences are buoyant, and have been increasing steadily for a number of years;
- the volume of Research Council support for research in the mathematical sciences does not reflect the above picture, whether the comparison is made with cognate disciplines in the UK or with mathematical sciences worldwide.

[1] http://www.cms.ac.uk/irm/irm.pdf
[2] Mathematics Today 42 pp. 132–134 and 168–169 (2006)

2. UK Mathematics - size and scale:

The volume of research in the mathematical sciences in the UK showed significant increases between 2000 and 2007, reflected both in the numbers of researchers submitted to the relevant Research Assessment Exercise[3] (RAE) subpanels, (with an overall increase of 9%) and in the level of Research Council grant income[4] :

Subject (UoA)	2000 FTE staff	2007 FTE staff	RCUK Research Income (£M)	RCUK Research Income (% increase)[5]	PhDs awarded
Pure	661	707	41	68%	625
Applied	735	850	116	67.5%	956
Statistics[6]	387	389	49	48%	509
Math. Science (= P+A+S)	1783	1946	206	63%	2090

The increases seen above in grant income and in PhDs awarded occurred steadily throughout the period of the last RAE – for example, 277 PhDs were awarded in 2001, 392 in 2006.

The expansions of the last decade in RC-funded research activity and in numbers of researchers were accompanied by a parallel concentration of activity into a smaller number of larger departments:

[3] For RAE data, see http://www.rae.ac.uk/ The figures from columns 3, 4 and 6 are taken from the submissions to RAE2008.

[4] Research income and numbers of PhDs awarded, are both over the period fixed for the RAE, that is, from 01/01/2001 to 31/07/2007.

[5] This column shows the percentage increases in income recorded in RAE2008, from UK Research Councils to UK mathematical science departments, as compared with RAE2001. Caution is needed in interpreting these figures, since Full Economic Costing was introduced during this period.

[6] For brevity, UoA 22, Statistics and Operations Research, is abbreviated to Statistics throughout this document.

Subject (UoA)	Number of submissions in 2001	Number of submissions in 2008	% of researchers in largest 4 submissions	% of researchers in largest 8 submissions
Pure	47	37	26%	44%
Applied	58	45	25%	40%
Statistics	46	30	26%	46%

The above tables demonstrate clearly that:

- while the number of active researchers has been increasing, research in the mathematical sciences in the UK is being concentrated in fewer places.

Powerful forces underlie this development:

- Research Council Grant structures favour large units[7];
- it is becoming increasingly difficult for small departments to obtain DTA grants to support postgraduate research students;
- Research Council rules for Doctoral Training Centres favour large units.

A further reason is probably also important, but is more difficult to quantify: universities, under pressure to maximise research income, may focus limited funds in areas where external grant income is more plentiful, rather than basing investment decisions on educational or academic grounds, or having regard to the UK's strategic needs.

These pressures run counter to the high popularity (with both students and with employers) of undergraduate degrees in the mathematical sciences across the breadth of the UK[8]. The buoyant demand for highly trained young mathematical scientists makes it vital to maintain a wide geographic spread of top-quality mathematicians and statisticians across the UK's higher education institutions.

[7] Note for example the introduction by EPSRC of Platform grants in the mathematical sciences for 2010-11 – see http://www.epsrc.ac.uk/plans/funding/Pages/maths.aspx

[8] See Section 4 below.

3. UK Mathematics – research quality:

UK research activity in the mathematical sciences was rated very highly in RAE2008, with 50% or more of research activity[9] in each of the three relevant subject areas being rated as internationally excellent (3*) or world-leading (4*).
Two additional features are worthy of note in the table below:

- Performance is very similar across all three disciplines.
- Excellence is widespread throughout the research community, not being concentrated only in a small number of large submissions.

	%4*	%3*	%2*	%1*
Pure: all submissions	14	39	39	6
Pure: biggest 4 submissions	30	43	26	1
Pure: biggest 8 submissions	22	39	30	6
Applied: all submissions	12	37	38	13
Applied: biggest 4	26	45	29	0
Applied: biggest 8	24	43	31	2
Statistics: all submissions	14	40	37	9
Statistics: biggest 4	22	45	29	4
Statistics: biggest 8	22	44	30	3

Citation indices provide a second indicator of the strength in depth of UK mathematical sciences in an international context at the start of the 21st century – the Thomson Reuters Citations Top 20, for mathematical sciences 1998-2008, shows Scotland in second place, England in fourth[10].

4. Comparison with sister disciplines.

The growth in the number of UK mathematical sciences researchers over the last decade has already been noted above. During the same period, activity in some cognate subjects has remained approximately constant or even declined. This is reflected both in the numbers of

[9] Averages in the table below are taken over submissions – that is, they are *not* weighted by the size of submissions.
[10] See http://www.timeshighereducation.co.uk/story.asp?sectioncode=26&storycode=406463 Denmark and the USA are respectively first and third.

academics in each discipline, and in the numbers of undergraduates produced, as the following table shows:

	FTE staff 2001	**FTE staff 2007**	**Graduating students (07-08)**
Chemistry	1300	1151	2860
Information Technology	1560	1910	????
Mathematical Sciences	1783	1947	5300
Physics	1668	1686	2515

There is a stark contrast between the levels of activity shown above and the amounts of Research Council funding. Thus, for example[11],

> *in 2008-9 EPSRC's total spend on research grants in the Mathematics Programme (which also covered Public Engagement) was £15.7M, whereas the corresponding figure for Physical Sciences was £97.8M, and that for Information Technology was £84.1M.*

Comparative allocations for 2009-10 show a similar pattern; here, for example, are the EPSRC programme allocations for mathematical sciences and for physical sciences:

EPSRC Programme 2009-10	**DTA[12] allocation**	**CDT[13] allocation**	**Responsive Mode**
Mathematical sciences	£11.1M	£10.8M	£14.3M
Physical sciences	£32M	5 CDTs – cost to find	£81.6M

A part of the mismatch shown in the above figures between RC support for mathematics and that for cognate subjects is a consequence of the fact that research in (for example) the physical sciences is more expensive than research in the mathematical sciences. However, even after the costs of equipment are removed, the imbalances are still striking – *evidence requested from EPSRC – need to add when available.*

[11] See http://www.epsrc.ac.uk/about/facts/Pages/budget.aspx
[12] DTA – Doctoral Training Account.
[13] CDT – Centre for Doctoral Training.

Other G8 countries do *not* accord the mathematical sciences the same Cinderella status as the UK. Thus, for example[14],

> *the NSF's 2010 budget for the mathematical sciences*
> *is $241M, while for physics it is $290M.*

5. Conclusions:

There is a vibrant mathematical science community in the UK, which

- produces a large annual cohort of well-trained graduating students in the mathematical sciences;
- helps to give the necessary mathematical training to those graduating in other disciplines, thereby crucially supporting the development of future scientists, engineers, financial and IT specialists;
- carries out a large volume of research of high international quality;
- produces around 400 PhDs per year in the mathematical sciences.

This activity is widely spread geographically; this indeed is a crucial feature, since the current numbers of undergraduate mathematicians, the support teaching for other subjects, and the variety of mathematical research will all be impossible to maintain if UK mathematical science is yet further concentrated into fewer centres.

[14] Source to be inserted. Need to get data for other countries.
NSF – USA National Science Foundation.